CHURCHLANDS SENIOR HIGH SCHOOL
MATHEMATICS SPECIALIST 3, 4 TEST TWO 2017
Calculator Section
Chapters 3, 4,

Name______________________________

Time: 15 minutes
Total: 14 marks

1. [9 marks: 2,1,2,4]

The position vectors of A and B, t hours after 10 am are $\boldsymbol{r} = -4\,\boldsymbol{i} - 4\boldsymbol{j} + t(2\boldsymbol{i} + 3\boldsymbol{j})$ and $\boldsymbol{r} = 3\,\boldsymbol{i} + 10\boldsymbol{j} + t(a\boldsymbol{i} + \boldsymbol{j})$ respectively.

a) Find **AB** t hours after 10 am.

$AB = OB - OA$

$= 3i + 10j + t(ai+j) - [(-4i-4j) + t(2i+3j)]$

$= [7+(a-2)t]i + (14-2t)j$ ✓✓

or $= \begin{pmatrix} 7+(a-2)t \\ 14-2t \end{pmatrix}$

b) Find in terms of a and t, the distance between A and B, t hours after 10 am.

Distance between A & B $= |AB|$

$= \sqrt{[7+(a-2)t]^2 + [14-2t]^2}$ ✓

$\sqrt{245 + 14at - 84t - 4at^2 + 8t^2 + a^2t^2}$

c) Explain why when collision between A and B occurs, $\boldsymbol{AB} = 0\boldsymbol{i} + 0\boldsymbol{j}$.

When collision occurs, the A & B are in the same position.
That is OA = OB.
Hence, $AB = OB - OA = 0i + 0j$ ✓✓

d) Find the value of a if the two particles never collide.

For the particles to collide $AB = 0i + 0j$

Hence $[7+(a-2)t]i + (14-2t)j = 0i + 0j$ ✓

$7 + (a-2)t = 0$ and $14 - 2t = 0$

$\Rightarrow t = 7$ ✓

Substituting in $7 + (a-2)\times 7 = 0$

$\Rightarrow a - 2 = -1$

$\Rightarrow a = 1$ ✓ Hence, for A & B not to collide, $a \neq 1$ ✓

2. [5 marks]
Find the parametric and hence the Cartesian equation of the line perpendicular to the vector $3i - 7j$ and passing through the point (-9,12).

vector equation of required line is

$$r = \begin{pmatrix} -9 \\ 12 \end{pmatrix} + \lambda \begin{pmatrix} 7 \\ 3 \end{pmatrix} \checkmark \qquad \begin{pmatrix} 3 \\ -7 \end{pmatrix} \cdot \begin{pmatrix} 7 \\ 3 \end{pmatrix} = 0 \checkmark$$

$$= \begin{pmatrix} -9 + 7\lambda \\ 12 + 3\lambda \end{pmatrix} \checkmark$$

hence parametric equations are
$$\begin{cases} x = -9 + 7\lambda \\ y = 12 + 3\lambda \end{cases} \checkmark$$

$$+ \begin{cases} -3x = 27 - 21\lambda \\ 7y = 84 + 21\lambda \end{cases}$$

$$-3x + 7y = 111$$

or $7y = 3x + 111$ ✓

or $y = \frac{3}{7}x + \frac{111}{7}$.